Phil 2310 Assignment 9
Fall 2010

This homework is due by the beginning of class on Mon, Nov 15th. Note that there are two pages to the homework.

Part I:
Chap 11: Multiple Quantifiers
Read pages 300-311.
Do 11.20

Part II:
Chap 14: More about Quantification
Read 364-378.
Do 14.1-14.3, 14.10-14.13

Part III:
Also translate the following sentences into FOL

1. Tom defeated at least two members of Team A.
2. Tom defeated at most one member of Team A who defeated Mary.
3. There is exactly one member of Team A who defeated both Tom and Mary and this person was undefeated. [[Note this does not imply that there is exactly one person who is undefeated]].

Part IV:
Prove these sequents. For these problems you may use FO con for two uses: DeMQ and NI (introducing the negation of an identity claim)

1. $\exists x(P(x) \wedge \forall y(x \neq y \rightarrow R(x,y))) \vdash \forall x(\neg P(x) \rightarrow \exists y(y \neq x \wedge R(y,x)))$
2. $\exists x \forall y(x=y \rightarrow P(x)), \forall x \forall y((P(x) \wedge P(y)) \rightarrow x=y) \vdash \exists x(P(x) \wedge \neg \exists y(P(y) \wedge x \neq y))$
3. $\forall x R(x,x), \exists x \exists y \exists z(R(x,y) \wedge R(y,z) \wedge \neg R(x,z)) \vdash \exists x \exists y \exists z(x \neq y \wedge x \neq z \wedge y \neq z)$
4. $\exists x \forall y(x=y), \neg \forall x P(x) \vdash \forall x \neg P(x)$

Part V:
Diagrams:
Determine which of these sentences are true on which of these diagrams (on the next page). For example, a 4x5 grid of 20 true/false answers is one way to answer this. It might help to think about students passing tests.

1. $\exists x(S(x) \wedge \forall y(T(y) \rightarrow P(x,y)))$
2. $\forall x(T(x) \rightarrow \exists y(S(y) \wedge \neg P(y,x)))$
3. $\forall x(T(x) \rightarrow \exists y \exists z(S(y) \wedge S(z) \wedge P(y,x) \wedge \neg P(z,x)))$
4. $\forall x \forall y((S(x) \wedge S(y) \wedge x \neq y) \rightarrow \exists z(T(z) \wedge P(x,z) \wedge P(y,z)))$
5. $\exists x \exists y(T(x) \wedge T(y) \wedge \forall z(S(z) \rightarrow (P(z,x) \vee P(z,y))))$

Part V (continued)

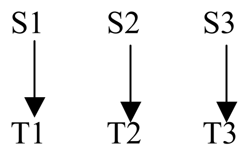
Diagram 1


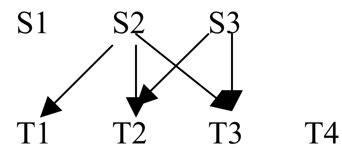
Diagram 2


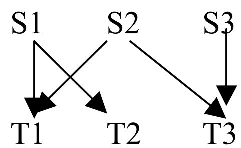
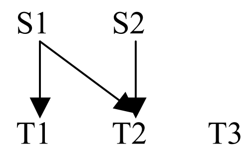
Diagram 3


Diagram 4


Part VI:
Diagrams as Models:

Show that each of the following arguments is invalid by producing a countermodel. In each problem, you should produce a single diagram where each of the premises is true but the conclusion is false. So produce three diagrams for this part. It might help to think about teachers attending meetings.

P1. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$
P2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$
Conc. $\exists x(M(x) \wedge \forall y(T(y) \rightarrow A(y,x)))$

P1. $\forall x(T(x) \rightarrow \exists y\exists z(x \neq y \wedge M(y) \wedge M(z) \wedge A(x,y) \wedge A(x,z)))$
P2. $\forall x(M(x) \rightarrow \exists y(T(y) \wedge A(y,x)))$
Conc. $\forall x\forall y((T(x) \wedge T(y) \wedge x \neq y) \rightarrow \exists z(M(z) \wedge A(x,z) \wedge A(y,z)))$

P1. $\exists x\exists y(M(x) \wedge M(y) \wedge x \neq y \wedge \forall z(Tz \rightarrow (A(z,x) \leftrightarrow A(z,y))))$
P2. $\forall x(T(x) \rightarrow \exists y(M(y) \wedge A(x,y)))$
Conc. $\forall x(T(x) \rightarrow \exists y\exists z(y \neq z \wedge M(y) \wedge M(z) \wedge A(x,y) \wedge A(x,z)))$